Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Differential Calculus #17

Summary of Differential Calculus

I. Ideas:

The derivative of a function is the rate of change of that function with respect to the variable.

The derivative is used to build dynamic mathematical models.

The derivative of $f(x)$ at a point $x = a$ is:

1. The rate of change of f at $x = a$
2. The slope of the tangent to the graph of f at $x = a$.
3. Formally:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

II. Notations: $\frac{df}{dx} = f'(x)$

III. Rules:

Here a, b are constants and f, g are functions.

Constants and addition: $(af + bg)' = af' + bg'$

Product: $(fg)' = f'g + fg'$

Quotient: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Chain: $\Big(f\big(g(x)\big)\Big)' = f'(g(x))\, g'(x)$

IV. Key Limits

$$\lim_{h\to 0}\frac{\sin h}{h}=1$$

$$\lim_{h\to 0}\frac{e^h-1}{h}=1$$

V. Methods

Implicit differentiation
Logarithmic differentiation

VI. Key Derivatives

f	f'
c, constant	0
x^m	mx^{m-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
e^x	e^x
$\log x$	$\frac{1}{x}$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$

VII Frequent Flyers

f	f'
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
x^x	$(1+\log x)x^x$

VIII Standard Applications

i. Newton's method of approximating roots
ii. Optimization
iii. Curve analysis
iv. Related Rates
v. L'Hospital's Rule for limits
vi. Exponential models:
   a. Radioactive decay
   b. Unconstrained growth
   c. Thermal flux – Newton Cooling